Ionospheric Tomography I:
Fundamentals of tomographic imaging of the ionosphere and its applications to radio propagation.
Summary

Introduction to tomography
Ionospheric structure
Radio waves in the ionosphere
Measuring total electron content
Tomographic reconstruction
Advantages, limitations and verification
Applications to radio systems
Tomography

Godfrey Hounsfield
Nobel Prize Winner

EMI CAT Scanner

X-ray Geometry

Image of Brain
Tomographic Imaging

- Obtain image from its projections
- Line integrals along intersecting ray paths
- Mathematical ideas long understood
- Development of computers in 1960s
- CAT scanners
- Successes in medical diagnostics
- Applications to geophysics
- Radio tomography of ionosphere
- First developments in USA and Russia
- New experimental technique
Ionospheric Tomography

- Use radio signals from satellites
- Receive at chain of ground stations
- Measure line integral of electron density (TEC)
- TEC along intersecting ray paths
- Invert data sets in reconstruction algorithm
- Obtain 2D image of electron density
- Large-scale spatial structure of ionosphere
Radio Tomography using LEO Satellites

- NIMS – Navy Ionospheric Monitoring Satellites
- up to six satellites, formerly NNSS
- circular polar orbits at 1100 km altitude
- chain of ground receiving stations
- measure total electron content along ray paths
Radio Tomography - Reconstruction

Total Electron Content along Ray Paths = Geometry of Path / Pixel Intersections × Electron Densities in Pixels

Measured Calculated from Satellite and Station Locations Unknowns
Tomographic Image
Image of Spatial Structure in Ionosphere

Tomographic Image: 14/12/96 10:46 UT
Electron Density (x10^{11} \text{ m}^{-3})

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Spatial Structure in Ionosphere

- Why develop radio tomography?
- Most experimental techniques give time series
- Tomography gives spatial images
- Wide-area cover from limited ground stations
- Remote and inaccessible regions

- Why is ionosphere structured?
Spatial Structure – Ionospheric Basics

- Vertical profiles of electron density
- Variable horizontal structure
- Interaction of basic mechanisms

Continuity equation

$$\frac{\Delta N}{\Delta t} = q - L - \text{div}(Nv)$$

- Electron density rate change
- Transport
- Loss
- Production
Ionospheric Basics

- Production \((q)\)
  - solar euv radiation
    - atomic oxygen \((O^+)\) plasma in F2-layer
  - particle precipitation
    - impact ionisation
    - high latitudes
    - ionospheric storms
Ionospheric Basics

- Loss (L)
  - neutral atmosphere chemistry
    - molecular species (N₂)
    - reaction rates
      - temperature dependent
      - velocity dependent
  - composition changes
    - [O] / [N₂] ratio
    - production / loss processes
    - storms

![Graph showing rate coefficients vs. ion temperature and relative velocity.](image)
Ionospheric Basics

- Transport $\text{div}(Nv)$
- motion constrained by magnetic field
  - neutral winds
  - diurnal, seasonal, storm
  - diffusion
  - $O^+ \leftrightarrow H^+$ protonosphere
- electric fields - $E \times B$ convection
  - equatorial anomaly
  - high latitudes

Electron density at any place and time depends on the balance between many different processes

Ionosphere is structured spatially on many different scales
Horizontal Structure

- *Balance* between these many different processes
- Results in horizontal *structure* in ionosphere
- Need to understand this structure and its origins
- Important for radio systems – hf propagation, GPS corrections

Ionospheric tomography creates images of such structure

- How does it work?
- How is radio wave affected by ionosphere?
Basic Magneto-ionic Theory

How is radio wave affected by ionosphere?
Need **refractive index** for propagation of radio wave in ionosphere

Appleton Equation

\[ n^2 = 1 - \frac{\frac{Y_T^2}{1 - jZ - \frac{Y_T^2}{2(1 - X - jZ)}}}{\pm \left( \frac{Y_T^4}{4(1 - X - jZ)^2 + Y_L^2} \right)^{1/2}} \]

‘It is virtually impossible for an ordinary mortal to make much sense of the Appleton equation(s) in their full glory.’

*(Hunsucker and Hargreaves, 2003)*
Appleton Equation

\[ n^2 = 1 - \frac{X}{X + \frac{Y^2}{1 - jZ - \frac{Y^2}{2(1 - X - jZ)}} + \left(\frac{Y^4}{4(1 - X - jZ)^2} + Y^2_L\right)^{1/2}} \]

- \( X = \left(\frac{\omega_N}{\omega}\right)^2 \)
  
  where \( \omega_N = \left(\frac{Ne^2}{\varepsilon o m_e}\right)^{1/2} \) is plasma frequency

  \( X \) depends on electron density \( (N) \)

- \( Y = \frac{\omega_B}{\omega} \)
  
  where \( \omega_B = \frac{Be}{m_e} \) is electron gyrofrequency

  \( Y \) depends on magnetic field \( (B) \)

- \( Z = \frac{\nu}{\omega} \)
  
  where \( \nu \) is collision rate

Refractive index \( (n) \) of an ionised medium with electron density \( (N) \), a magnetic flux \( (B) \) and electron collision frequency \( (\nu) \)
Trans-ionospheric Propagation

The radio wave frequency ($\omega$) $>>$ plasma frequency ($\omega_N$), so that $X << 1$

Neglecting the magnetic field ($Y=0$) and collisions ($Z=0$) the refractive index now has a very simple form

$$n = 1 - X / 2 \quad \text{or} \quad n = 1 - N e^2 / 2 \epsilon_0 m_e \omega^2$$

Inserting values and using $f$ instead of $\omega$ for frequency

$$n = 1 - 40.3 N / f^2 \quad \text{(with } N \text{ in } m^{-3} \text{ and } f \text{ in Hz)}$$

Since $n < 1$, phase of wave in ionised medium will advance with respect to free-space propagation
Carrier Phase Advance and Doppler Shift

After travelling a distance $dl$ (ie $dl/\lambda$ wavelengths) phase of the wave has changed by

$$2\pi \frac{dl}{\lambda} = 2\pi f n \frac{dl}{c}$$

Thus over a path $l$ through the ionosphere the phase change will be

$$-(2\pi f/c) \int n \frac{dl}{c} = -2\pi f \frac{l}{c} + (2\pi \times 40.3 / cf) \int N \frac{dl}{l}$$

Change in phase of a wave travelling at the speed of light

• phase advance is *cumulative* along path
• depends on the *Total Electron Content (TEC)* along slant path

$$N_T = \int N \frac{dl}{l}$$

Numerically, *phase advance* (in radians) due to the ionosphere is

$$\varphi = (8.45 \times 10^{-7}) N_T / f \quad \text{(with } N_T \text{ in } m^{-2} \text{ and } f \text{ in Hz )}$$

Since *frequency* is rate of change of phase, ionosphere imposes *Doppler shift* on the wave
Measurement of TEC by Differential Carrier Phase Method

• In practice, measurement of phase requires a reference signal.

• One way in which this can be achieved is for the source to transmit two coherent frequencies derived from a common oscillator.

• Forms the basis of the Differential Carrier Phase or Differential Doppler technique used for tomography.
Differential Carrier Phase (Differential Doppler) Technique

Satellite transmits two coherent frequencies $f$ and $pf$, where $p$ is constant (NIMS satellites used for tomography transmit on 150MHz and 400MHz so that $p = 8/3$)

Compare received phase of lower frequency with that of the higher frequency divided by $p$

$$\Delta \phi = \phi_f - \phi_{pf} = \{-2\pi f l/c + 2\pi x 40.3 N_T/f c\} - \{(1/p)\{-2\pi pf l/c + 2\pi x 40.3 N_T/pf c\}\}

The first and third terms cancel because they represent the phase changes for free space propagation of the two signals along the path

Thus the differential phase shift due to the ionospheric TEC is

$$\Delta \phi = \{1 - 1/p^2\} (8.45 \times 10^{-7}) N_T/f$$

Can measure relative phase accurately but still have $2\pi$ ambiguity to solve to get absolute TEC.
Absolute TEC from Differential Carrier Phase Measurements

- With two or more stations can match the observation in the region of overlap to give the same vertical TEC
- Hence obtain *absolute* TEC measurements versus latitude
Slant TEC and Vertical TEC

- **Slant TEC**
  \[ N_T = \int N \, dl, \text{ where } l \text{ is a slant ray path from satellite to ground} \]

- **Vertical TEC** (needed for comparisons and models)
  \[ N_T = \int N \, dh, \text{ where } h \text{ is a vertical path through ionosphere} \]

- **Equivalent Vertical TEC**
  In practice, most measurements are slant TEC but are converted to the vertical using an assumed thin-shell ionosphere at a chosen mean height (\( dl = dh \sec \chi \)) giving equivalent vertical TEC.

- Most references to TEC are in fact to equivalent vertical TEC.

- Measure TEC in TEC units, where 1 TECU \( \equiv 10^{16} \text{ m}^{-2} \)
Ionospheric Tomography

\[
\text{Total Electron Content along Ray Paths} \times \text{Geometry of Path / Pixel Intersections} = \text{Electron Densities in Pixels}
\]

Measured
\[
\text{Calculated from Satellite and Station Locations}
\]

Unknovns
Basics of ionospheric tomography - pixels

For all ray paths between satellite and ground stations $M$ such simultaneous equations

In matrix notation:

$$ y = Ax $$

$y$ - slant total electron content measurements

$x$ - unknown electron densities

The problem of ionospheric tomography is to solve this equation to find the electron densities
Tomographic Algorithms - Iterative

- Early algorithms used *Row Action Methods*
  - *Iterative* solutions
  - Successive corrections to an assumed initial
    *Background Ionosphere*
  - Need background because of incomplete information
  - No horizontal ray paths

- *ART (Algebraic Reconstruction Techniques)*

After $k$ iterations the set of electron densities is given by

$$x_{k+1} = x_k + \lambda_k \frac{y_i - (x_k, a_i)}{\|a_i\|_2} a_i$$

where

$$\|a_i\|_2 \equiv (a_i, a_i) = \left(\sum_j A_{ij}^2\right)$$

with $a_i$ representing the $i$ th row of $A$ and $\lambda_k$ a relaxation constant $< 1$
Tomographic Algorithms – Direct Inversion
Discrete Inverse Theory (DIT)

- Many different mathematical techniques have been used by different workers

- Finding an appropriate **background ionosphere** to initialise any algorithm is the **key to ionospheric tomography**

- Use extension of DIT method of Fremouw et al. (1992 and 1994) to obtain **background ionosphere**

- Described by linear combination of a number of **basis functions** with different **weightings**
First Stage of Reconstruction

To obtain *basis functions*

- **vertical form:** Chapman profiles of E and F regions
  - spanning complete range found in data set of
    - peak heights,
    - scale heights and
    - scale-height gradients in topside
- **horizontal structure:** Fourier series with power-law taper

- **Use truncated Singular Value Decomposition (SVD) to obtain set of Empirical Orthonormal Functions (EOFs) to form basis.**

- **Background ionosphere can be described by linear combination of these with different weightings**
Tomography Algorithms – Discrete Inverse Theory

Background ionosphere described by linear combination of a number of basis functions with different weightings

\[ b = B x \]

- \( b \) - the electron density values in pixels,
- \( B \) - basis functions representing ionosphere
- \( x \) - weight given to each basis function

Using \( H \) - geometry matrix of path/pixel intersections

\[ y = H B x \]

Thus

\[ y = H B x \] or \[ y = A x \]

where now \( A = H B \)
Need to solve \( y = A x \)

- Reformulate to avoid need for *absolute calibration* of measured TEC
- Use \( y \) as *differences* in TEC between successive ray paths, that is, use *relative TECs*
- Solve to find \( x \) - *the weight given to each basis function in the linear combination that best describes the background ionosphere, consistent with the measured TEC*

\[
\hat{x} = x_0 + C_{x_0x_0} A^T \left( C_{y_0y_0} + AC_{x_0x_0} A^T \right)^{-1} \left( y_0 - Ax_0 \right)
\]

\[
\hat{C}_{xx} = C_{x_0x_0} - C_{x_0x_0} A^T \left( C_{y_0y_0} + AC_{x_0x_0} A^T \right)^{-1} AC_{x_0x_0}
\]
Second Stage of Reconstruction

2. Find smaller -scale structure:
   - Iterative second stage uses background ionosphere generated by the first stage as starting condition for algebraic reconstruction technique (ART) algorithm.

   - Image Grid:
     - Altitude: 25 km    Latitude: 0.25 degree

   - Can use method to incorporate other types of ionospheric measurements - for example, ionosonde data
Radio Tomography: Advantages and Limitations

Advantages
- New experimental technique
- Spatial images of large-scale density structures
- Wide coverage from limited ground stations
- Complementary role to other instruments

Limitations
- Understood at early stage
- Limited-angle technique, no horizontal ray paths
- Incomplete information on vertical structure
- Temporal coverage dependent on satellite orbits
Does it work?
Experimental station chains used by UWA group

UK

Scandinavia

Svalbard
Does it work?
Verification using EISCAT radar

Figure 6.10. Tomographic image of electron density for the pass at 21:36 UT, using 20 ray paths per degree movement of the antenna (upper plot) and the electron-density contour from the EISCAT BLOB scan between 21:02 and 21:28 UT (lower plot) on 15 October 1993.
Differences in Layer Height

Kersley et al. (1997)
Radio Tomographic Imaging:
Applications to practical radio systems

- Complementary to ionosonde measurements
- Validation of ionospheric models
- Mapping of ionospheric parameters
- Oblique sounding
- HF ray tracing
- HF direction finding
- Space weather
Tomography and Testing of Empirical or Parameterised Models

Dabas and Kersley (2003)
Tomography and Coupled Thermosphere Ionosphere Plasmasphere Model (SUCTIP)

Idenden et al. (1999)
Radio Tomographic Imaging: Applied to mapping of ionospheric parameters

Maps of peak electron density (NmF2) over Europe

Dabas and Kersley (2003)
Validation of foF2 Maps

• Tomography from UK stations + Chilton ionosonde
• Validation using ionosondes near trough and at mid-latitudes
Validation of Maps using Ionosondes

- Use of tomographic image gives better agreement with ionosonde foF2 than any of the models alone
- Potential for nowcasting
Tomography and Oblique Ionograms

- Profiles at mid-point show good agreement in F-layer
- Tomographic images may help in assessment of assumptions needed for oblique ionogram reduction

Heaton et al. (2001)
Tomography and HF Ray Tracing

Estimation of Maximum Usable Frequency (MUF)

- Tomography with ionosonde input gave smallest errors in estimation of MUF(F2)
- Better than FAIM, PIM and IRI-95 models
- Climatological model better for E-layer MUF

Rogers et al. (2001)
Tomography and HF Direction Finding

Figure 1. Map illustrating the path from Uppsala to Leicester.

Figure 3. Measurements made at 10.39 MHz between 20 November 2001 (day 324), 1200 UT to 23 November 2001 (day 327), 1200 UT. The panels represent, time-of-flight (ms, top left), Doppler shift (Hz, top right), azimuth of strongest mode (°, bottom left), and elevation of strongest mode (°, bottom right).

Warrington et al. (2002)
Radio Tomographic Imaging: applied to HF DF

TEC plots vs latitude from UK tomography chain

- Trough wall supporting HF propagation
Tomography and Vertical TEC
Other forms of ionospheric ‘tomography’

- Statistical imaging of ionospheric irregularities
- GPS imaging

TEC map
- Follow temporal changes

Electron density image
- Limited height resolution
Conclusions

Radio Tomographic Imaging

- versatile new experimental technique
- large-scale spatial structure
- wide area coverage from few ground stations
- applications to applied radio science
- applications to geophysical research
- footprints of space-weather processes